Methodics of valuation of expanded uncertainty

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This valuation of the uncertainty with asymmetric limits of the confidence interval in relation to the mean value of sound level, what gets out of the fact of qualifying of the mean value as average of the energy.

1. Average sound level

The average sound level (the odds identically probable events or measurements) in count the as so called „logarithmic average" definite by the equation:

$$L_{av.} = 10 \cdot \lg\left[\frac{1}{n}\sum_{i=1}^{n} 10^{\frac{L_i}{10}}\right] \qquad \text{[A]}$$

2. Sound level definition

The sound level L_p expressed in decibels is defined as 10 common logarithms from the relation of the square of the sound pressure to the square of the reference pressure 2 *10^{-5} Pa:

$$L_p = 10 \cdot \lg\frac{p^2}{p_0^2}, \;\; dB \qquad \text{[B]}$$

where p_0 - reference pressure of 2*10^{-5} Pa (the threshold of the hearing for 1000 Hz)

3. Average relative exposition

Transforming equations [A] and [B] one finds that:

$$\frac{p_{śr.}^2}{p_0^2} = \frac{1}{n}\sum_{i=1}^{n}\frac{p_i^2}{p_0^2} \qquad \text{[C]}$$

that is to say the **value awaited** for the magnitude $\mathbf{p^2/p_0^2}$ - the relative (for the given time of the measurement) exposition - definite with the example on „the arithmetical mean" are for which definite mathematical statistical examples. Marking the magnitude $\mathbf{p^2/p_0^2}$ as E we receive:

$$E_{śr.} = \frac{1}{n}\sum_{i=1}^{n} E_i \qquad \text{[D]}$$

4. The A type uncertainty

The uncertainty of the type A (marked as U_A) refers the statistical scattering of results of the measurement treated as random variables about following features:

- the identical probability of the event - for the value awaited definite as the arithmetical mean,
- independent,
- repeatable,
- the measurement does not bear on the result.

4.1. Uncertainty of the result of average from the measuring series

The A type uncertainty of the **imission level** or the **acoustic background level** on the base of **n** performed elementary measurement counts according to following rules:

- results of elementary measurement of the sound level expressed in „dB” we transform to the form of the relative exposition (from the example [B]):

$$E_i = \frac{p_i^2}{p_0^2} = 10^{\frac{L_i}{10}} \qquad \text{[E]}$$

- we count the mean value (the value awaited) - after the example [D]:

$$E_{śr.} = \frac{1}{n}\sum_{i=1}^{n} E_i$$

- we count estimator of the standard-average deviation **s** of the mean value in accordance with the example:

$$s = \sqrt{\frac{1}{n\cdot(n-1)}\sum_{i=1}^{n}\left(E_{śr.} - E_i\right)^2} \qquad \text{[F]}$$

- we value the uncertainty on confidence level 95%, taking the Student's distribution (the coefficient **τ(n)** for the two-sided section, i.e. $t_{0,975}$ for n-1 degrees of freedom):

$$U_{A,95}(E_{śr.}) = \tau(n)\cdot s \qquad \text{[G]}$$

- we qualify the **confidence interval** on the level 95% for the **relative exposition** as:

$$\left\langle \left[E_{av.} - U_{A,95}(E_{av.})\right] \leftrightarrow \left[E_{av.} + U_{A,95}(E_{av.})\right] \right\rangle \qquad \text{[H]}$$

- we counts the limits of confidence interval, definite as above, expressing it in values of the A-weighted sound level (on the base of the example [B]):

$$\left\langle 10 \cdot \lg\left[E_{av.} - U_{A,95}(E_{av.})\right] \leftrightarrow 10 \cdot \lg\left[E_{av.} + U_{A,95}(E_{av.})\right]\right\rangle \quad [I]$$

in other words:

$$\left\langle L_{lower\,limit} \leftrightarrow L_{upper\,limit} \right\rangle \quad [J]$$

- we count values of the uncertainties of the average value for the sound level:

  the upper value $+U^{+}_{A,95}$ (the uncertainty for greater values from average) and low value $-U^{-}_{A,95}$ (the uncertainty for smaller values from average) definite in relation to the average value for the sound level for the higher calculated confidence interval as:

$$\left\langle \left[L_{av.} - U^{-}_{A,95}(L_{av.})\right] \leftrightarrow \left[L_{av.} + U^{+}_{A,95}(L_{av.})\right] \right\rangle \quad [K]$$

  and we express the result as:

$$\mathbf{L_{av.}\ (+U^{+}_{A,95};\ -U^{-}_{A,95})} \quad [L]$$

4.2. Uncertainty of the result of calculation of emission

The A-type uncertainty of **emission sound level** on base of calculated values of average imission sound levels and acoustic background levels counts according to following rules:

- we count the value of emission in term of relative exposition:

$$E_{em} = E_{im} - E_{background} \quad [M]$$

  Remark: expressing above as sound levels, we obtain well-known equation:

$$L_{em} = 10 \cdot \lg(E_{em}) = 10 \cdot \lg(E_{im} - E_{background}) = 10 \cdot \lg(10^{0,1 \cdot L_{im}} - 10^{0,1 \cdot L_{background}})$$

- we qualify the uncertainty of the result of emission for the relative exposition:

$$U_{A,95}(E_{em}) = \sqrt{\left[U_{A,95}(E_{im})\right]^2 + \left[U_{A,95}(E_{background})\right]^2} \quad [N]$$

- then we repeat the procedure according to examples from [H] to [L].

ATTENTION:

In the case of the difference among the imission level and with the background level above 10 dB in compliance with the methodics of measuring [4] can be skipped the influence of the acoustic background. However then one ought to take into account the connected error with such reduction (the inflatedness of the result of emission) which carries out 0,5 dB for above mentioned difference of 10 dB, and 0,1 dB for the difference 15 dB.

4.3. Uncertainty of the result of calculation of equivalent sound level

4.3.1. Uncertainty of the qualification of time

The uncertainty of the qualification of the duration of the acoustic situation for which one performed elementary measurement, one ought to qualify according to following rules:

- the event through the all normative time of the observation **T** - the uncertainty is equal „0",
- events about the compactly definite duration in the normative time of the observation - the uncertainty equal „0",
- events about the variable duration in the normative time of the observation - the uncertainty qualified in compliance with guidelines contracted in [3] for the model „of the rectangle", with where lower and the upper limit of the interval of time events determines properly of minimum and maximum time of the situation, and the mean time of the duration is accepted as the arithmetical mean from these limits.
  Uncertainty on confidence level 95% is taken as 95% of maximal deviation from the average value into every side.
- in the case of acoustic situations whose durations assumed other schedules - one ought to apply the individual approach.

4.3.2. Uncertainty of the qualification of equivalent sound level

The A-type uncertainty of the equivalent sound level are calculated on base of prior calculated interests of emission E_{em_k} for every acoustic situation and time of duration of that situation together with suitable uncertainties $U_{A,95}(E_{em\_k})$ and $U_{A,95}(t_k)$.

- the equivalent relative exposition for **m** of acoustic situations in the normative observation time **T** one qualifies after the example for relative expositions:

$$E_{eq} = \sum_{k=1}^{m} \frac{t_k}{T} \cdot E_{em\_k} = \sum_{k=1}^{m} E_{eq\_k} \qquad [O]$$

Remark: expressing above as sound levels, we obtain well-known equation:

$$L_{eq} = 10 \cdot \lg(E_{eq}) = 10 \cdot \lg(\sum_{k=1}^{m} \frac{t_k}{T} \cdot E_{em\_k}) = 10 \cdot \lg(\sum_{k=1}^{m} \frac{t_k}{T} \cdot 10^{0,1 \cdot L_{em\_k}})$$

- there one qualifies the uncertainty of the equivalent relative exposition for every acoustic situation:

$$U_{A,95}(E_{eq\_k}) = \sqrt{\left[\frac{t_k}{T} U_{A,95}(E_{em\_k})\right]^2 + \left[10^{0,1 \cdot L_{em\_k}} \cdot \frac{U_{A,95}(t_k)}{T}\right]^2} \qquad [P]$$

- we qualify the uncertainty of the result of the equivalent relative exposition for sum of all acoustic situations

$$U_{A,95}(E_{eq}) = \sqrt{\sum_{k=1}^{m} \left[U_{A,95}(E_{eq\_k})\right]^2} \qquad [Q]$$

- then we repeat the procedure according to examples from [H] to [L].

5. The B type uncertainty

The B-type uncertainty (marked as U_B) is connected with the inaccuracy of gauges, procedures of investigative and accepted models of acoustic occurrences.

Manners of qualifying of this uncertainty cause that „*the valuation of the uncertainty of the type B is more the experimental art than the craft*" [2] and we qualify her with other methods than the mathematical statistics - a base for these respects they are:

- certificates, certificates,
- literature data ,
- earlier obtained measuring-data ,
- the own experience and the knowledge,
- the detailed acquaintance of investigated occurrences.

For given uncertainties for sound levels, from the example [B] we receive examples on the uncertainty for relative expositions (in the general case):

- upper limit of uncertainty for relative exposition:

$$\Delta E_+ = 10^{\frac{L_{av.}+\Delta L_+}{10}} - 10^{\frac{L_{av.}}{10}} = 10^{\frac{L_{av.}}{10}} \cdot \left(10^{\frac{\Delta L_+}{10}} - 1 \right) \quad \text{[R1]}$$

- lower limit of uncertainty for relative exposition:

$$\Delta E_- = 10^{\frac{L_{av.}}{10}} - 10^{\frac{L_{av.}-\Delta L_-}{10}} = 10^{\frac{L_{av.}}{10}} \cdot \left(1 - 10^{\frac{\Delta L_-}{10}} \right) \quad \text{[R2]}$$

Because appears here the dependence from the current value of the level of the sound, then more comfortable the form of the presentation of the B-type uncertainty will be the relative uncertainty:

- upper limit of relative uncertainty for relative exposition:

$$\frac{\Delta E_+}{E} = 10^{\frac{\Delta L_+}{10}} - 1 \quad \text{[S1]}$$

- lower limit of relative uncertainty for relative exposition:

$$\frac{\Delta E_-}{E} = 1 - 10^{\frac{\Delta L_-}{10}} \quad \text{[S2]}$$

For given / established / estimated B-type uncertainties expressed as deviations of sound levels we receive following relative B-type uncertainties on confidence level 95% for relative expositions, under the assumption that deviations, upper and lower, for sound levels are equal:

- for the occurrence for which the probability ditribution is fashioned as „rectangle”:

$$(\Delta L_+ = \Delta L_- = \Delta L) \Rightarrow \begin{cases} \dfrac{U^+_{B,95}(E)}{E} = \left(10^{\frac{\Delta L}{10}} - 1\right) \cdot 0{,}95 \\ \dfrac{U^-_{B,95}(E)}{E} = \dfrac{U^+_{B,95}(E)}{E} \cdot 10^{\frac{-\Delta L}{10}} \end{cases} \qquad [T]$$

- for the occurrence for which the probability ditribution is fashioned as „triangle”:

$$(\Delta L_+ = \Delta L_- = \Delta L) \Rightarrow \begin{cases} \dfrac{U^+_{B,95}(E)}{E} = \left(10^{\frac{\Delta L}{10}} - 1\right) \cdot \underbrace{\left(1 - \dfrac{\sqrt{5}}{10}\right)}_{\approx 0{,}767} \\ \dfrac{U^-_{B,95}(E)}{E} = \dfrac{U^+_{B,95}(E)}{E} \cdot 10^{\frac{-\Delta L}{10}} \end{cases} \qquad [U]$$

- the qualification of the eventual relative B-type uncertainty for **W** taken into account factors demands executions of bills on relative expositions separately for the upper limit and for the lower limit:

$$\frac{U^+_{B,95}(E)}{E} = \sqrt{\sum_{j=1}^{w} \left[\frac{U^+_{B,95}(E_j)}{E}\right]^2} \qquad [V1]$$

$$\frac{U^-_{B,95}(E)}{E} = \sqrt{\sum_{j=1}^{w} \left[\frac{U^-_{B,95}(E_j)}{E}\right]^2} \qquad [V2]$$

- Having eventual relative uncertainties for relative expositions one can also qualify values of the eventual (upper and lower) uncertainty for sound levels:

$$U^+_{B,95}(L) = 10 \cdot \lg\left(1 + \frac{U^+_{B,95}(E)}{E}\right) \qquad [W2]$$

$$U^-_{B,95}(L) = 10 \cdot \lg\left(1 - \frac{U^-_{B,95}(E)}{E}\right) \qquad [W2]$$

6. Expanded uncertainty

The expanded uncertainty for confidence level 95% ($U_{R,95}$) is result of the scattering of results of measurement of the investigated noise together with the acoustic background (imission) and the acoustic background, and the connected inaccuracy with the put-upon measuring-equipment and applied with the measuring-procedure:

$$U_{R,95} = \sqrt{U_{A,95}^2 + U_{B,95}^2}$$

where

$U_{A,95}$ - the A-type uncertainty connected with the scattering of results of the measurement
$U_{B,95}$ - the B-type uncertainty connected with the equipment and the measuring-procedure

- the qualification of the expanded uncertainty demands executions of bills on relative expositions separately for the upper limit and for the lower limit:

$$U_{R,95}^{+}(E_{eq}) = \sqrt{\left[U_{A,95}^{+}(E_{eq})\right]^2 + \left[U_{B,95}^{+}(E)\right]^2} \qquad [X1]$$

$$U_{R,95}^{-}(E_{eq}) = \sqrt{\left[U_{A,95}^{-}(E_{eq})\right]^2 + \left[U_{B,95}^{-}(E)\right]^2} \qquad [X2]$$

- then we repeat the procedure according to examples from [H] to [L] and we receive the result expressed as:

$$\mathbf{L_{eq}\ (+U^{+}_{R,95};\ -U^{-}_{R,95})}$$

7. The literature:

- [1] "*Statystyka w pomiarach akustycznych - podstawy*"
  ["*Statistics in Acoustic Measurements - the basics*"]
  Mikołaj Kirpluk
  - paper published on the conferece „XXXIV Winter School of Vibroacoustics Hazards" (February 2006)
- [2] „Statystyka dla fizyków"
  [„The statistics for physicists"]
  Roman Nowak
  (Wydawnictwo Naukowe PWN, Warszawa 2002) ISBN 83-01-13702-9
- [3] „Wyrażanie niepewności pomiaru. Przewodnik."
  [„The expression of the uncertainty of the measurement. The guide."]
  (GUM, 1999) ISBN 83-906546-1-x
- [4] schedule no 8 to regulation of Ministry of Enviroment dated 23.12.2004 *on requirements in the range of the leadership of measurement of the emission* (State Journal of 2004 no. 283, item 2842)